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A Heuristic Approach for Antenna Positioning in Cellular Networks*

MICHEL VASQUEZ
LGI2P, EMA-EERIE, Parc Scientifique G. Besse, 30035 Nîmes cedex 01, France
email: michel.vasquez@site-eerie.ema.fr

JIN-KAO HAO
LERIA, Université d'Angers, 2 Bd Lavoisier, 49045 Angers cedex 01, France
email: jin-kao.hao@univ-angers.fr

Abstract

The antenna-positioning problem concerns finding a set of sites for antennas from a set of pre-defined candidate sites, and for each selected site, to determine the number and types of antennas, as well as the associated values for each of the antenna parameters. All these choices must satisfy a set of imperative constraints and optimize a set of objectives. This paper presents a heuristic approach for tackling this complex and highly combinatorial problem. The proposed approach is composed of three phases: a constraint-based pre-processing phase to filter out bad configurations, an optimization phase using tabu search, and a post-optimization phase to improve solutions given by tabu search. To validate the approach, computational results are presented using large and realistic data sets.

Key Words: large scale combinatorial optimization, tabu search, radio network planning

1. Introduction

The planning process of mobile radio networks may be roughly divided into two different problems: the Antenna Positioning Problem (APP) and the Frequency Assignment Problem (FAP). The basic APP is concerned with a series of decisions, such as the site locations for the antennas, the number and types of antennas for each site, and the associated values for the antenna parameters. The FAP has to do with the assignment of a set of available frequencies to the antennas of the network. Both problems involve a great deal of constraints, and they are closely related, because a good (bad) antenna positioning may make frequency assignment easier (harder).

Until now, many studies have been carried out for the FAP and highly effective optimization algorithms have been developed; see for instance, (Box, 1978; Crompton, Hurley and Stephen, 1994; Duque-Anton, Kunz and Ruber, 1993; Funabiki and Takefuji,

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1992; Hao and Dorne, 1995; Hao, Dorne and Galinier, 1998; Hurley, Thiel and Smith, 1996; Lai and Coghill, Jaumard et al., 2000). Many network operators now routinely use frequency-planning tools integrating such algorithms.

On the contrary, studies on optimization algorithms for the antenna-positioning problem seem much more limited. Indeed, most existing studies are oriented towards small-scale micro-cellular or indoor systems involving only several antennas (Fortune et al., 1995; McGeehan and Anderson, 1994; Sherali, Pendyala and Rappaport, 1996). Other studies focus on optimizing some antenna parameters or some specific objective such as the coverage of a (relatively small) area (Calégari et al., 1996; Molina, Athanasiadou and Nix, 1999). No real optimization algorithm is available yet for antenna positioning and optimization of large-scale radio networks. Tasks related to antenna positioning are essentially carried out with the help of engineering tools integrating some simulation functions, which leads to largely sub-optimal solutions.

With the continuous and rapid growth of communication traffic, large scale planning becomes more and more difficult and cannot be realized in an optimal or near optimal manner. Automatic or interactive optimization algorithms and tools would be very useful and helpful. Advances in this area will certainly lead to important improvements concerning the service quality in terms of coverage and interference and allowing the decrease of the installation cost. The APP thus constitutes a significant stage in the process of cellular network planning.

The general antenna-positioning problem can be informally described as follows. Given a list of candidate sites for antennas, several types of antennas, and a discretized geographical working area characterized by a set of points with information related to traffic estimation and the radio threshold, the aim is to select some sites among the candidate sites, and for each selected site determine the number and types of antennas, as well as the associated values for each of the antenna parameters. All these decisions must satisfy a set of imperative constraints (cover, handover, one connected-component cell) and optimize a set of objectives (number of sites used, amount of traffic that can be handled, level of potential interference, efficiency of transmitters). It is easy to see that the problem is highly combinatorial. The number of possible combinations is enormous for realistic networks, leading to search spaces as large as $2^{4,000,000}$.

The heuristic approach we develop is composed of three sequential phases: a constraint based pre-processing phase to eliminate a large number of "bad" combinations, an optimization phase by tabu search working in a reduced search space, and a post optimization phase by fine tuning of antenna parameters.

This approach is applied to two large and realistic test data sets corresponding respectively to an urban network and a highway network in a GSM system. Experimentation shows that the proposed approach is highly effective, robust, and flexible.

2. Problem description

In this section, we give the basic elements necessary for the general understanding of the antenna-positioning problem. A more detailed presentation of the APP can be found in

Table 1. Characteristics of a real network data set.

	Area width	Area length	RTP	STP	TTP	Sum of traffic	Candidate sites
Urban network	46.5 km	45.8 km	56792	17393	6652	2988,08 Erlangs	568

Reininger (1997) and Reininger and Caminada (1998a). A cellular network is composed of three entities: a discretized geographical *working area*, where signals and traffic are measured, *mobile (cellular) stations (MS)*, which define the services, and *antennas*, which can be placed on some pre-defined sites within the geographical area.

2.1. Working area

The geographical working area on which a network is deployed is discretized into a finite number of points called reception test points (RTP). For each RTP, a radio signal is tested. From the set of RTP, two other sets are defined:

- the set of service test points (STP), where the radio signal must be higher than a threshold Sq to allow the establishment of communications (Section 2.2),
- the set of traffic test points (TTP), for each of which the traffic of communication measured in Erlang is estimated.

The traffic implies the communication, a TTP is thus necessarily a STP and the following relation is always verified:

$$\{\mathrm{TTP}\} \subset \{\mathrm{STP}\} \subset \{\mathrm{RTP}\}$$

The working area is also described by a list of pre-defined candidate sites on which antennas may be placed.

Table 1 and figure 1 summarize all these concepts. This example corresponds to an urban area of 49.6 km $\times$ 45.8 km.

The mesh step for the discretization is 200 meters. We thus have $248 \times 229 = 56792$ RTP.

2.2. Mobile station (MS)

A network provides a service for a category of mobile stations. A quality threshold, noted Sq hereafter, defines this service. A network may provide different services, thus different quality threshold. If the radio signal at a given point of the working area is higher than the required Sq, then the cellular phones that are at this point can communicate. The value of the threshold Sq is dependent on the MS considered and expressed in decibel (dBm) (Table 2).

Table 2. Examples of thresholds per service.

Mobile station	Sq in dBm
2 Watt incar	−78
2 Watt outdoor	−84
8 Watt outdoor	−90

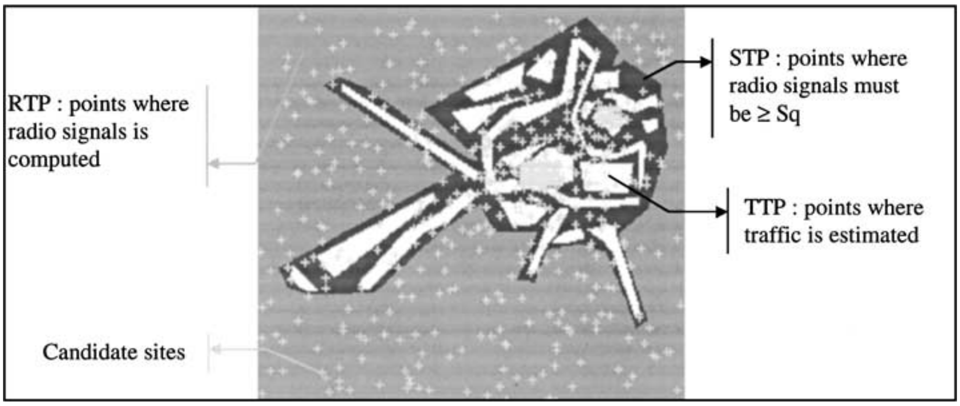

Figure 1. A real network working area and its candidate sites.

An MS has another specific characteristic that must be considered: the reception sensitivity of the MS, or mobile sensitivity (Sm). Sm has an average value of −99 dBm, however, a signal of this value is not sufficient for an MS to establish communication with an antenna, but it does scramble an already-established communication. This point will be re-examined later when we evoke the noise level of a network.

2.3. Antennas

In general, there are several types of antennas available in a network, characterized primarily by their transmission gain (Gs) and their propagation diagrams (figure 2). In this work, we consider 3 types of antennas: omnidirectional (OMNI), large directional (LD), and small directional (SD).

The principal parameters of these antennas are:

- the power, PS, which can vary from 26 to 55 dBm,
- the azimuth (for a directional antenna) between 0° and 360°,
- the tilt (for a directional antenna) between −15° and 0°,

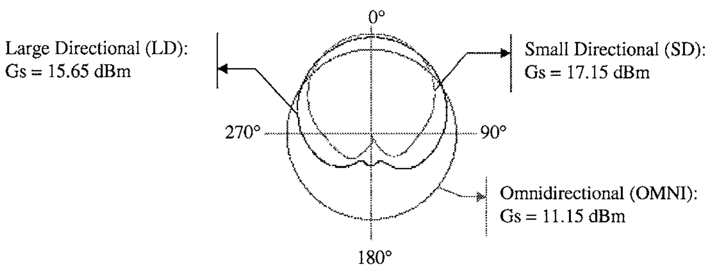

Figure 2. 3 types of antennas.

- the number of transmitters (TRX) assigned to the antenna for a given traffic. In a GSM system, a conversion table determines this number according to the material used. Table 3 shows such an example where an antenna may require 1 to 7 TRX (thus 1 to 7 channels). Note that the number of TRX is directly determined by the traffic and does not need to be tuned by the optimization algorithm.

These antennas can be placed on pre-defined candidate sites in the working area. In our case, a site can host either one OMNI antenna or one to three LD or SD antennas.

2.4. Base station and cell

A *base station*, BS b, is defined by a quintuplet b = (site, antenna, tilt, azimuth, power). It corresponds thus to the choice of a site, an antenna on this site and the parameter values of the antenna. For example, for the above network, the BS b = (356, LD, 0, 30, 38) corresponds to the placement of a LD antenna on the site numbered 356. This antenna has a tilt of 0°, an azimuth of 30°, and a power of 38 dBm.

Other components are also involved in the definition of a BS, such as BS transmitter loss and BS receiver sensibility (Reininger, 1997; Reininger and Caminada, 1998a), and the same applies to the MS (Section 2.2). Since these values are constant for a given situation, they will not be further discussed in this paper.

In order to assess the signal quality at each point, a radio wave propagation model is needed. Such a model is able to predict the propagation loss of an electromagnetic field between a site and each RTP of the working area. To compute the prediction, the model takes into account the site coordinates, its height, the RTP coordinates, the set of obstacles between the site and the RTP (buildings, mountains...), and the angle of incidence between the site and the RTP.

Table 3. Number of transmitters and traffic capacities.

TRX	1	2	3	4	5	6	7
Erlang	2.9	8.2	15	22	28	35.5	43

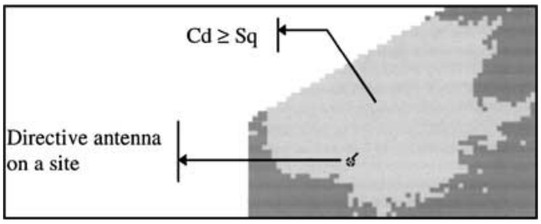

Figure 3. Cell corresponding to b = (356, LD, 0, 30, 38).

We evoked above only downlink signals emitted by base stations towards cellular phones. In fact, it is also necessary to take into account signals from MS towards BS (uplink signals). It is, however, shown in Reininger and Caminada (1998a) that if the downlink signal, coming from a BS, is higher than the quality threshold Sq and the uplink signal is stronger than the downlink signal (which is indeed the case in GSM systems), then it is not necessary to be concerned with uplink signals.

Thus, starting from the data of a BS in a network we will be able to calculate, for each point of the geographical area, a radio signal, noted hereafter as Cd. The *cell* of a BS corresponds thus to the set of STP covered by the BS, i.e. for which the signal received from this BS is the best one and higher than the quality threshold Sq. Figure 3 illustrates the link between an isolated BS and its cell.

Since radio wave propagation is never homogeneous and isotropic, the cell of a BS is always irregularly bounded, depending on the topography and the transmitting power. Moreover, the cell of a BS is dependant on other BS emitting from overlapping areas.[1]

2.5. *Constraints*

Each STP must be served by at least one BS. Therefore, the union of the cells in a given network must be equal to the set of all the STP located in the working area. This necessity constitutes the global *coverage* constraint for a network.

When an MS moves from one cell to another, the network must be able to guarantee the continuity of the communication. To accomplish this, it is essential that each cell has a nonempty intersection (handover area) with its neighboring cells. This requirement constitutes the *handover* constraint, which must be respected by all the cells of the network.

The STP contained in a cell may constitute several connected components. Connected components play a significant (and negative) role in the quality of a network (Reininger and Caminada, 1998b): the more connected components there are for a cell, the more interference there may be. Also, cells having more connected components make it difficult to manager the handover. Therefore, one of the constraints of the APP is that each cell of the network constitutes only one connected component. This local constraint is called *one connected component (OCC)* constraint, for which in this paper, only components containing more than 8 STP are taken into account (see Section 3.3). For example, the cell in figure 3 satisfies the OCC constraint, even if, in addition to the main connected component, the cell has one component of 2 STP and six other components of 1 STP.

2.6. Objectives

The installation of a new site is usually very expensive for the network operator. For this reason, a major objective of APP is to minimize the number of sites used.

A complete network is made up of a certain number of cells (typically one hundred for the networks we studied). Each STP receives signals coming from several BS. These overlapping signals are necessary for the purpose of handover, but at the same time generate interfering noise. Therefore, a second important objective is to minimize the level of noise within the network.

A cell covers a certain number of traffic test point TTP. However, given that the total traffic served by a cell cannot exceed 43 Erlang (see Table 3), it is possible that the traffic of some TTP within a cell may not be totally served. Therefore, a third objective is to maximize the total traffic supported by the network.

One notices from Table 3 that the closer the traffic of a cell is to the maximum accepted by a given number of TRX, the better the output of these TRX will be. This leads to a fourth objective, which is to maximize the traffic yield of the BS transmitters in the network.

The preceding classification of constraints and objectives corresponds to a particular scenario that was used within the framework of this study. Of course a network operator can interpret all these concepts differently and exchange some constraints and objectives. Moreover, other constraints and objectives may be introduced.

The above constraints and objectives are rather interdependent of each other, and often have conflicting natures. First, the coverage constraint is opposed to the objective of minimizing the number of sites used. Second, the handover constraint implies the existence of several signals at one same point, and can increase the level of noise which one wants to minimize. Third, in order to maximize the amount of traffic the network can handle, one needs to limit the size of each cell. Now in order to guarantee coverage, one need to increase the size or the number of cells. In both cases, the increase is accompanied by an increase in the level of noise, as well as an increase in difficulty of managing the OCC constraint. Finally, one notices also that it is not easy to jointly satisfy the OCC constraint and the coverage constraint. This last observation is important, since it implies the difficulty of producing feasible solutions for the APP.

In the previous discussion, we have presented in the most general way the concepts that highlight the antenna-positioning problem. The problem thus consists in choosing, among all the possible BS, a set of BS which satisfies the coverage, handover, and OCC constraints, while minimizing the number of sites used, maximizing the ensured traffic and yield of the transmitters, and minimizing the noise level.

Now, if one considers cells rather than BS, the APP can also be seen as a coverage problem of a plane surface: one wishes to cover the surface (working area) with various forms of cells with multiple constraints between these forms, while optimizing the objectives.

3. Formulation of problem

In this part, the mathematical model for the APP used in this work is presented. The details of this model can be found in Reininger (1997), Reininger and Caminada (1998a, 1998b). The model shown here reflects only a particular scenario. Other models are surely possible. However, the basic idea of the heuristic approach presented in this paper may be applied to other scenarios.

3.1. Basic notations

- *ST* set of all the service test points STP in the working area,
- Sq service threshold defined by a power value for a given station (Table 2),
- Sm cellular phone station receiver sensitivity defined by a power value,
- *TT* set of the traffic test points TTP of the working area: $TT \subset ST$,
- Ps antenna power,
- BS quintuplet (site, antenna, tilt, azimuth, Ps),
- *BS1* set of selected BS that correspond to a network design,
- $\mathrm{Cd}_{\mathrm{b,p}}$ field strength received at a STT p $\in$ *STP* from a BS b $\in BS1$,
- *L* set of the candidate sites for a given network,

The positioning of an antenna corresponds to the choice of a finite number of BS, denoted by $BS1$, chosen among all possible ones.

For each b belonging to $BS1$ we define its cell Cell(b) as follows:

$$Cell(b) = \{\mathrm{p} \in ST/\mathrm{Cd}_{\mathrm{b,p}} \geq \mathrm{Sq} \text{ and } \forall \mathrm{b}' \in BS1\ \mathrm{b}' \neq \mathrm{b}\ \mathrm{Cd}_{\mathrm{b,p}} > \mathrm{Cd}_{\mathrm{b',p}}\}$$

The second part of this definition is important. It indicates that the cell of a BS depends not only on this BS but also on the other BS in the network.

3.2. Coverage constraint

All the STP of the working area must be covered by an antenna. This constraint is formally expressed by the following formula:

$$ST = \bigcup_{b \in BS1} Cell(b) \tag{1}$$

3.3. One connected component (OCC) constraint

Each cell defined by a BS b must have only one connected component. If we define $\mathrm{C_b}$ the number of connected components of *Cell(b)*, this constraint is expressed by the following

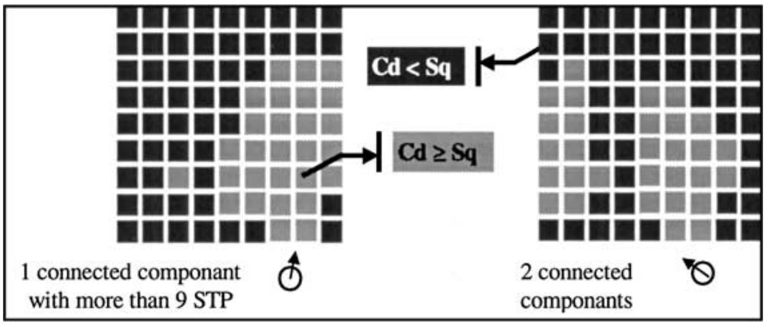

Figure 4. Connected components of a single cell.

formula:

$$\forall b \in BS1\ C_b = 1 \tag{2}$$

In this work, we do not take into account components containing fewer than MINC STP. MINC is an integer parameter to be fixed. In this study, MINC = 9 is used.[2] Figure 4 illustrates this principle.

One notices that the OCC constraint would be difficult to satisfy if the coverage constraint is taken into account at the same time. Indeed, when one adds a BS or increases the size of a cell to get a larger cover, one may "cut" a one CC cell into two cells or create a cell of multiple components.

3.4. Handover constraint

The handover area of a cell is defined by the set of STP p covered by the BS b, such that there is at least one other BS b′, from which the field strength $Cd_{b',p}$ on p is greater than the threshold Sq, and at most 7 dBm above or below the field strength $Cd_{b,p}$ received from the BS b, or:

$$\begin{aligned}\mathrm{handCell(b)} = \{&p \in \mathrm{Cell(b)}/\exists\, b' \in BS1 \text{ and } Cd_{b',p} \geq Sq \text{ and}\\ &|Cd_{b,p} - Cd_{b',p}| \leq 7\mathrm{dBm}\}\end{aligned}$$

The handover constraint, which requires a non-empty handover area for each cell, is expressed by the following formula:

$$\forall b \in BS1\ \mathrm{handCell(b)} \neq \emptyset \tag{3}$$

One notices that the model does not take into account the location and the number of handover points (Reininger and Caminada, 1998a). This definition corresponds in fact to a weak form of the handover requirement (number of minimal handover point = 1 per cell) and may be easily extended to include more than one minimal handover points. Computational simulations show that this weak form of handover is sufficient to ensure good handover in a network when the coverage constraint is satisfied. This observation may be interpreted as an indicator that the coverage constraint implies somewhat handover. We observe also that the handover constraint defined by (3) is satisfied as soon as there are a sufficient number of cells in the network.

3.5. Minimize the number of sites used

This objective is defined by:

$$\min \sum_{i \in L} c_i \times y_i, \quad y_i = \begin{cases} 1, & \text{if site } i \text{ is selected} \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

c_i is the cost of site i. In this paper, we suppose all the sites have a unit cost:[3]

$$\forall i \in L, \quad c_i = 1$$

This restriction corresponds to networks in construction. There are, however, networks in extension for which the cost of a site depends on the operation that one carries out: creation of a new site, modification or suppression of an existing site in the initial network. We will discuss this point in the conclusion section and show that the resolution approach presented in the paper remains valid in this situation.

3.6. Minimize the noise level

Noise level estimation is not straightforward. If there is too much overlap between cells, noise level will be very high. We have defined a cell as the set of STP with the best signal coming from the same BS b. So $\mathrm{Cd}_{\mathrm{b,p}}$ is the best signal received at a given point p of the cell Cell(b). Ideally, each STP of Cell(b) should not receive more than h signals lower than $\mathrm{Cd}_{\mathrm{b,p}}$ and greater than the required sensitivity threshold Sm (Section 2.2.). These h signals are used for handover. In our work, h value is fixed at 3, but it is a parameter that can be varied according to the model used. Signals after the hth and greater than Sm are considered as noise. For each point p of Cell(b), consider the sorted list of signals greater than Sm:

$$\mathrm{Cd}_{\mathrm{b,p}} \geq \mathrm{Cd}_{\mathrm{b1,p}} \geq \cdots \mathrm{Cd}_{\mathrm{bh,p}} \geq \cdots \mathrm{Cd}_{\mathrm{bk,p}} > \mathrm{Sm}$$

Hence the noise level at point p is given by:

$$\Upsilon(p) = \sum_{h < j \leq k} Cd_{bj,\ p} - Sm \quad \text{(k is dependent on p)}$$

The objective of minimizing the total amount of noise is expressed as follows:

$$\min \sum_{p \in ST} \Upsilon(p) \tag{5}$$

3.7. Maximize the amount of traffic of the network

The total traffic a BS b can handle is given by the following formula:

$$\text{traffic\_BS (b)} = \sum_{p \in TT \cap Cell(b)} \text{traffic\_point(p)}$$

According to this value, one will assign a number of transmitters TRX to this station by using the conversion Table 3. If the total traffic required by the TTP of a BS exceeds 43 Erlang, then the exceeding traffic may be lost. It is for this reason that we introduce the concept of the traffic hold of a cell:

$$\text{trafficHold(b)} = \begin{cases} \text{traffic(b)} & \text{if (traffic(b)} \leq 43), \\ 43 & \text{otherwise.} \end{cases}$$

The objective of maximizing the amount of traffic hold of a network is expressed by:

$$\max \sum_{b \in BS1} \text{trafficHold(b)} \tag{6}$$

3.8. Maximize traffic yield

Given the traffic hold of a BS b and the traffic capacity of b (see Section 2.3 and Table 3), we define the traffic yield for a cell by:

$$\text{trafficYield(b)} = \frac{\text{trafficHold(b)}}{\text{trafficCapacity(b)}}$$

Hence, the objective of maximizing the traffic yield is expressed by the following formula:

$$\max \sum_{b \in BS1} \text{trafficYield(b)} \tag{7}$$

4. Problem analysis

This section presents the main characteristics of the APP, allowing us to have an idea about the difficulty of the problem. These characteristics are: a very high number of search combinations, a high complexity of computation, and a high requirement of memory.

4.1. Large number of combinations

The values of the parameters of antennas were discretized as follows:

- Ps $\in$ [26..55] and δPs = 2 dBm $\rightarrow$ |Ps| = 15,
- azimuth $\in$ [0..359] and δazimuth = 10° $\rightarrow$ |azimuth| = 36,
- tilt $\in$ [−15..0] and δtilt = 3° $\rightarrow$ |tilt| = 6.

These values were considered to be sufficient for the precision of calculations and the resolution of the problems. This quantification is a first step towards reducing the number of search combinations.

Thus, an omnidirectional antenna has |Ps| = 15 possible settings, and a directional one has |Ps| × |azimuth| × |tilt| = 3240 possible settings. Thus, to put a BS at a site, we have $15 + 3240 + 3240 = 6495$ possible choices (denoted by $|\mathrm{BS}_{\mathrm{site}}|$). If $|L|$ represents the number of candidate sites of a network number of candidate sites, we get $|L| \times |\mathrm{BS}_{\mathrm{site}}|$ possible choices for a BS in the network.

To build a network is to find a combination of base stations, among the possible $|L| \times |\mathrm{BS}_{\mathrm{site}}|$ ones, which satisfies all the constraints and optimizes the objectives. We thus have $2^{|L| \times |\mathrm{BSsite}|}$ potential choices of configurations, even if a large number of them are not feasible.

For example, the network of figure 1 has 568 candidate sites, and thus a search space of $2^{568 \times 6495} = 2^{3,689,160}$ combinations.

4.2. Computational complexity

For the purpose of clarity and conciseness, we did not evoke all the computation rules for calculating electromagnetic fields. These rules, given in Reininger (1997), are primarily trigonometrical formulas of angles between STP and sites. A priori, an optimization process has to check at each stage that all the constraints involved are satisfied, and to count those that are violated. For the OCC and handover constraints, the computing complexity generated by this task is about $|BS1| \times |ST|$, where $|BS1|$ represents the number of BS selected at a given stage of the optimization process.

Cell management, which is essential for the representation of most of the constraints and certain objectives, is very expensive to compute. Indeed, on the one hand, it is necessary to calculate the signals emitted by all the selected BS on all the STP, and to sort these values for each STP, in order to determine the cells associated with the best fields, and, on the other hand, to calculate the noise level and indicate the other fields higher than Sm.

For an average of 100 selected BS, the network of figure 1 requires about 100 × 17393 non-trivial calculations (arctang, real divisions, sorting of Cd, calculation of connected components) to evaluate a configuration. This requires more than one million non-elementary operations.

Table 4. Data for the APP.

Set of candidate sites: L	$\lvert L \rvert \sim 500$
Set of RTP: R	$\lvert R \rvert \sim 100000$
Set of STP	From 10000 to 100000
Set of TTP	From 5000 to 100000
Propagation loss matrix	$\lvert L \rvert \times \lvert R \rvert$
Angle of incidence matrix	$\lvert L \rvert \times \lvert R \rvert$

4.3. Memory consumption

Computing the signals dynamically using a radio propagation model is very time consuming, and, therefore, cannot be used during an optimization process. Propagation loss data are thus pre-computed and stored in a propagation loss matrix where propagation loss has been predicted from each site to each RTP. Associated to these values we have an incidence matrix that gives the incidence angle for each couple (site, RTP). For each type of antenna, we also have the horizontal and vertical diagrams. Using this data, one can compute the field strength $\mathrm{Cd}_{\mathrm{b,p}}$ by using the formulas detailed in Reininger (1997). Table 4 gives an idea about the quantity of data necessary for the problems that we solved.

Typically, the data concerning the radio signal, the values of traffic, the coordinates of the sites, and the points of a network require more than 200 MB of memory.

5. General heuristic approach for the APP

The APP is thus highly combinatorial and very difficult to resolve. This remains true even for finding feasible solutions satisfying all the constraints. In particular, it is not at all obvious how the OCC and coverage constraints can be satisfied simultaneously.

To tackle the APP, we have developed a heuristic approach, which is composed of three sequential phases: a pre-processing phase based on a filtering principle, an optimization phase based on tabu search, and a post-optimization phase by fine tuning antenna parameters.

The pre-processing phase uses some *filtering* criteria to eliminate or filter out many undesirable base stations (or cells) that cannot contribute to a good solution. We calculate, site by site and antenna by antenna, all the possible cells generated by each BS (site, antenna, power, tilt, azimuth). According to the filtering criteria, we decide for each cell whether the cell is kept or rejected. For example, if the filtering criterion used is the OCC constraint, then any cell violating this constraint will be definitively eliminated. Similarly, if we want to limit the size of the cells, we may use this criterion to filter out the cells exceeding the desired size. Therefore, this pre-processing step allows us to greatly reduce the number of combinations of the search space. For network such as the one we used, this step retains typically 200,000 to 400,000 BS, from some 4,000,000 possible ones. Let us notice another important point: Computations of field strengths for each point in the working area are carried out at this phase and are no longer necessary during the optimization phase which is carried out by tabu search.

From the set of BS produced by the pre-processing phase, the optimization phase by tabu search will construct solutions by choosing a subset of BS that satisfy all the constraints of the problem and optimize the objectives. To do this, the tabu algorithm, starting with an empty solution, tries to extend at each iteration its current solution by adding a BS and dropping some existing BS, if necessary, (for instance, to continue satisfying the OCC constraint). The choice of which BS is added at each iteration takes into account the objectives, and checks that the coverage constraint is satisfied.

Finally, the post-optimization phase is applied to improve the solution produced by the tabu algorithm. This phase can be used to optimize objectives or repair the rare constraints that remain unsatisfied. Post-optimization is realized by the fine-tuning of antenna parameters.

6. Pre-processing

6.1. Constraint based pre-processing

As previously mentioned, one of the main difficulties of the APP concerns the management of the OCC and coverage constraints. One well-known technique for constraint handling in general is the penalty-based approach. In this approach, constraints are considered as objectives and integrated into a weighted evaluation function:

$$f' = \sum_{i=1}^{i=m} f_i + \sum_{j=1}^{i=n} p_j \times \Phi(c_j)$$

where:

- f_i represents one initial objective,
- p_j is a penalty to be defined for constraint c_j,
- $\Phi(c_j)$ equals 1 if c_j is satisfied, equals 0 otherwise.

An advantage of this approach lies its flexibility, while its main drawback is the difficulty in fine tuning the penalties. Indeed, if some constraints are incompatible and hard to satisfy, these constraints may never be satisfied. This is precisely the case for the OCC and coverage constraints.

To cope with this difficulty, we introduce a special technique for handling the OCC constraint (the global coverage constraint is handled with the penalty approach, see Section 7.2). The basic idea is to use the OCC constraint in an active way to filter out "bad" BS which violate this constraint, and which consequently cannot contribute to a good solution. Only "good" BS are retained.

Recall that a candidate site can host one omnidirectional (OMNI) antenna or one to three directional (LD or SD) antennas, which results in 6495 potential base stations. For a given site, all its BS configurations are not of equal interest. In particular, a BS whose cell has many connected components can in no way be useful for a final solution due to the OCC constraint. Therefore, it would be beneficial to eliminate such BS from the search space from the beginning. That is what we do during the pre-processing phase. For every possible BS b = (site, antenna, power, tilt, azimuth) of every candidate site, we carry out all the necessary computations of field strengths to calculate the corresponding cell of the BS, and then count the number of its connected components having more than 9 STP (see Section 3.3). If the number of connected components is greater than one, i.e. the OCC constraint is violated, then the cell is not counted. Otherwise, the cell is recorded in a data structure together with all related information. Therefore, the left cell in figure 4 (Section 3.3) is kept, while the right one is rejected.

To calculate the connected components, we use the "scan line blob coloring algorithm", which is well known in the field of computer vision (Ballard and Ballard, 1982). This algorithm scans the working area from top left to bottom right and labels STP belonging to the same cell with the same color. To accomplish this, it considers four points around the current one: the three neighboring points on top and the left neighboring point in an 8-neighborhood. For a single BS, this algorithm has a time complexity of $O(|Cell(b)|)$.

This OCC constraint-based pre-processing phase allows one to significantly reduce the size of the search space, especially in the situations where many irregular obstacles are present in the terrain. Indeed for the network of figure 1, this filtering step retains only 294000 BS. The combinations in our search space are thus reduced from $2^{3689160}$ (intractable) to 2^{294000} (tractable).

The idea behind the pre-processing is very general and other criteria, like the noise level and the traffic, can be easily used separately or conjointly for this pre-processing phase. Such pre-processing techniques were implemented and experimented upon in our study. However, we are unable to describe them further within the framework of this paper.

Therefore, the pre-processing phase offers great flexibility, allowing us to generate many different search spaces with different characteristics, which can then be used by the optimization phase to produce various solutions. This flexibility represents a nice feature for multi-objective optimization problems such as the APP.

6.2. *Connectivity constraint transformation*

After this filtering stage, we have cells which satisfy the OCC constraint individually, and which have additional proprieties when other filtering criteria are applied. Since the OCC is difficult to handle, this constraint must remain satisfied during the tabu optimization phase, which consists in adding and dropping BS. For this purpose, we divide each cell into two parts, called the "kernel" and "border," and introduce a new constraint called the "kernel constraint."

Let δSq be a dBm value greater than 0: $\delta\text{Sq} > 0$ dBm. For each cell, one considers the 2 following sets:

$$\text{kernel}(b) = \{p \in Cell(b)/Sq + \delta Sq \leq Cd_{b,p}\}$$
$$\text{border}(b) = \{p \in Cell(b)/Sq \leq Cd_{b,p} < Sq + \delta Sq\}$$

Figure 5 illustrates this partition.

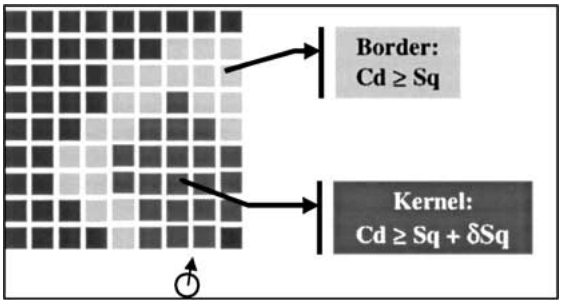

Figure 5. Cell = {border} ∪ {kernel}.

Then the kernel constraint states that the kernels of two different cells do not overlap:

$$\forall (b, b') \in \mathit{BS1} \times \mathit{BS1}, \quad b \neq b' \Rightarrow \text{kernel}(b) \cap \text{kernel}(b') = \varnothing$$

Notice that the partition of a cell into kernel and border may be adjusted by the value given to δSq. By varying the value of δSq, we can make the kernel constraint stronger or weaker.

Now, during the tabu optimization phase, this kernel constraint is used so that the OCC constraint will remain satisfied. Therefore, the management of the OCC is replaced by handling this simpler kernel constraint.

The kernel constraint does not forbid the overlapping of the border zone of one cell with that of another cell. Such an overlapping zone is typically used to ensure the handover constraint.

Let us now consider a more detailed example. Table 5 shows a partial solution involving 4 BS. Figure 6 gives the cells of these BS (left) together with their kernel and borders (right). In this example, $\delta\text{Sq} = 4$ dBm is used to defined the border areas. One notices that the overlap of the two adjacent cells concerns only their borders.

In summary, the pre-processing step generates, from the raw data of the problem, a reduced set of BS, as well as their representation in terms of kernel and border. The next step consists in constructing a solution from these BS.

Table 5. A partial configuration for the urban network.

Site	Antenna	Tilt	Azimut	Ps
131	LD	0	90	46
356	LD	0	30	38
397	LD	–6	300	46
493	SD	–6	90	50

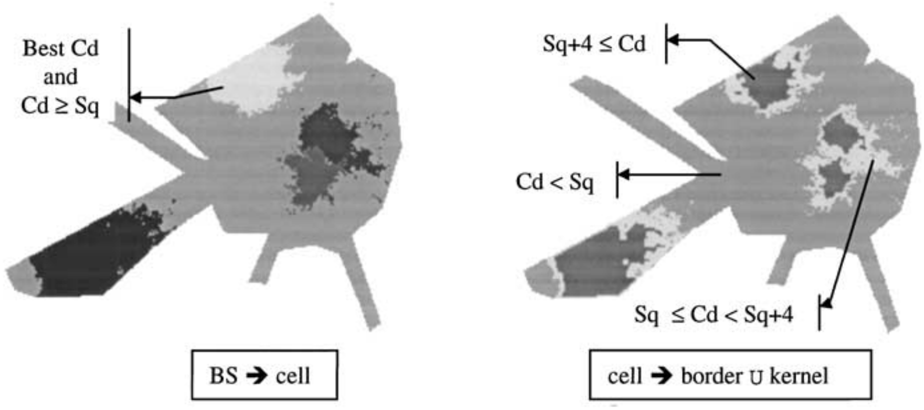

Figure 6. Four cells (left) and their kernel-border representation.

A solution will be designed by putting together some BS in such way that all the STP of the working area are covered, the kernel constraint is respected, each cell shares a handover area with some other cells, and the objectives are optimized. In practice, the handover constraint is automatically satisfied if a sufficiently large number of BS is present in a solution and if all STP are covered. The remaining task is essentially to satisfy the coverage constraint while optimizing the objectives, which is accomplished with a tabu search algorithm.

7. Optimization by tabu search

We now present the main ingredients of our optimization algorithm based on tabu search. For a complete presentation of TS, the reader is invited to consult the comprehensive book by Glover and Laguna (1997).

7.1. Configuration

Let ß be the set of BS selected by the pre-processing step. We define a first search space S to be the set of all possible binary vectors with |ß| component:

$$S = \{0, 1\}^{|ß|}$$

Let $s = (b_1, \ldots, b_{|ß|})$ be such a vector of S. Each component b_i identifies a particular BS in ß. If b_i equals 1 then the corresponding BS is retained in the partial solution, otherwise, the BS is rejected. The space S thus represents all the possible networks that can be built starting from ß.

However, one notices that many configurations of S are not of interest, since they do not even verify the rule of antenna placement on a site (one OMNI or 1 to 3 LD or SD per site, see Section 2.3). To translate this implicit constraint of the model we associate with each type of antenna a weight ρ:

$$\rho : \begin{cases} \rho(OMNI) = 3, \\ \rho(LD) = 1, \\ \rho(SD) = 1. \end{cases}$$

For a BS b we use ρ(b) to denote the value ρ (b antenna type) and define the following function:

$$As : S \times L \mapsto \{0, 1, \ldots\} \quad As(s, l) = \sum_{b=1 \text{ and } b \text{ on site } 1} \rho(b)$$

We now define the subspace $T \subset S$ verifying the rule of antenna placement on a site:

$$T = \{s \in S/\forall l \in L \quad As(s, l) \leq 3\}$$

It is clear that this reduced search space is of greater interest than the initial space S.

From T we now define a last search space $X \subset T$ that respects the kernel constraint (Section 6.2):

$$X = \left\{\left(b_1, \ldots, b_{|\text{ß}|}\right) \in T/\forall_i, \forall_j, i \neq j \text{ and } b_i = 1 \text{ and } b_j = 1 \\ \Rightarrow \text{kernel}(b_i) \cap \text{kernel}(b_j) = \varnothing\right\}$$

Therefore, the search space X includes all the configurations that satisfy both the rule of antenna positioning on a site and the kernel constraint. Since many non feasible configurations are excluded from X compared with the initial search space, we have $|X| \ll |S|$.

7.2. *Configuration evaluation*

In order to guide the tabu algorithm to visit the search space, one needs a function for evaluating the configurations. Since the APP involves multiple objectives and multiple constraints, the evaluation is somewhat complicated. In this work, we took a hierarchical approach to evaluate the configurations. Formally, for a given configuration s of X, it is evaluated by the following vector function.

$$\xi(\mathrm{s}) = \langle \mathrm{c}_0(\mathrm{s}), \mathrm{f}_1(\mathrm{s}), \mathrm{f}_2(\mathrm{s}), \mathrm{f}_3(\mathrm{s}), \mathrm{f}_4(\mathrm{s}) \rangle \text{ where:}$$

- $c_0(s)$ = coverage(s) = number of STP covered by the cells of s,
- $f_1(s)$ = trafficHold(s) = sum of traffic held by all the cells of s,
- $f_2(s)$ = noise(s) = sum of noise generated by each selected BS of s,
- $f_3(s)$ = number of sites where BS are installed,
- $f_4(s)$ = traffic Yield(s).

The first component c_0 of this evaluation function corresponds to the *coverage constraint*. This component takes priority over the other components (f_1, f_2, f_3 and f_4) that are related to the different objectives of the problem. A higher priority for the component c_0 helps to guide the search to find first feasible solutions. Another possibility would consider the component c_0 at the same level as the other objectives at the risk of never finding a feasible solution.

For the components f_1, f_2, f_3 and f_4, any priority order may be used according to the importance we give to each objective. For our presentation, we chose arbitrarily the following priority order P:

$$\mathrm{P}(\mathrm{f}_1) > \mathrm{P}(\mathrm{f}_2) > \mathrm{P}(\mathrm{f}_3) > \mathrm{P}(\mathrm{f}_4)$$

Given two configurations s1 and s2, s1 is said to be better than s2, denoted by $\xi(s1) > \xi(s2)$, if the following condition is verified:

$$\xi(s1) > \xi(s2) \Leftrightarrow \begin{cases} (\mathrm{c}_0(\mathrm{s}_1) > \mathrm{c}_0(\mathrm{s}_2)) \text{ or,} \\ (\mathrm{c}_0(\mathrm{s}_1) = \mathrm{c}_0(\mathrm{s}_2) \quad \text{and} \quad \mathrm{f}_1(\mathrm{s}_1) > \mathrm{f}_1(\mathrm{s}_2)) \text{ or,} \\ (\mathrm{c}_0(\mathrm{s}_1) = \mathrm{c}_0(\mathrm{s}_2) \quad \text{and} \quad \mathrm{f}_1(\mathrm{s}_1) = \mathrm{f}_1(\mathrm{s}_2) \quad \text{and} \quad \mathrm{f}_2(\mathrm{s}_1) < \mathrm{f}_2(\mathrm{s}_2)) \text{ or,} \\ \ldots \end{cases}$$

$\Delta\xi(\mathrm{s1,s2}) = \langle \Delta \mathrm{c}_0(\mathrm{s1,s2}),\ \Delta \mathrm{f}_1(\mathrm{s1,s2}),\ \Delta \mathrm{f}_2(\mathrm{s1,s2}),\ \Delta \mathrm{f}_3(\mathrm{s1,s2}),\ \Delta \mathrm{f}_4(\mathrm{s1,s2})\rangle$ denotes the vector variation of ξ.

We also use another function of evaluation: $\xi'(\mathrm{s}) = \langle \mathrm{c}'_0(\mathrm{s}), \mathrm{f}_1(\mathrm{s}), \mathrm{f}_2(\mathrm{s}), \mathrm{f}_3(\mathrm{s}), \mathrm{f}_4(\mathrm{s})\rangle$ where:

$$c'_0(s) = \sum_{p\text{covered}} w(p) \text{ where } w(p) \text{ is a weight value greater than 0, and}$$

$$\xi'(\mathrm{s}) \Leftrightarrow \xi(\mathrm{s}) \quad \text{if } w(p) = 1\ \forall\ \mathrm{p} \in ST.$$

We will see the usefulness of this evaluation function in Section 7.5.

7.3. *Neighborhood and move*

We now introduce the neighborhood function **N** over the search space X. More precisely, this function $\mathbf{N} : X \to 2^X$ is defined as follows.

Let $\mathrm{s} = (\mathrm{b}_1,\ \mathrm{b}_2, \ldots, \mathrm{b}_{|ß|}) \in X$ and $\mathrm{s}' = (\mathrm{b}'_1,\ \mathrm{b}'_2,\ \mathrm{b}'_{|ß|}) \in X$ then s′ is a neighbor of s, i.e. $\mathrm{s}' \in \mathbf{N}(\mathrm{s})$, if and only if the following conditions are met:

1) $\exists$! i such that $\mathrm{b}_i = 0$ and $\mathrm{b}'_i = 1$ $(1 \le \mathrm{i} \le |ß|)$
2) for the above i, $\forall\ j{\neq}i \in \{1 \ldots |ß|\}$ kernel $(\mathrm{b}_i) \cap$ kernel $(\mathrm{b}_j) \neq \emptyset \Rightarrow \mathrm{b}'_j = 0$

Thus, a neighbor of s can be obtained by *adding* a BS (flipping a variable b_i from 0 to 1) in the current configuration and then *dropping* some other BS (flipping some b_j from 1 to 0) to repair the kernel constraint violation. Consequently, a move *mv* to obtain a neighbor s′ from a configuration $s = (b_1, b_2, b_3 \ldots b_{|ß|})$ is characterized by a series of flipping operations:

b_i from 0 to 1

b_{i1} from 1 to 0

$\ldots\ldots$

b_{in} from 1 to 0

where $b_{i1} \ldots b_{in}$ are variables linked to b_i by the kernel constraint. That means that there is at least one same element (i.e. STP) in both kernel (b_i) and kernel(b_j) for $j \in J_i = \{i_1, \ldots, i_n\}$. Such a moved is denoted by $mv(i) = (b_i : 0 \rightarrow 1, b_{j1} \ldots b_{in} : \; 1 \rightarrow 0)$. Use $s' = s + mv(i)$ to denote the neighbor of s obtained by applying mv(i) to s.

It should be clear that from a configuration $s = (b_1, b_2, \ldots, b_{|ß|})$, there are as many possible moves as the number of variables in s having a value of 0.

Let $|s| = \sum_{1 \leq i \leq |ß|} b_i$, then s has exactly $|ß| - |s|$ neighboring configurations (i.e. $|\mathbf{N}(s)| = |ß| - |s|$).

7.4. *Tabu list management and aspiration criteria*

The main role of a tabu list is to prevent the search from short-term cycling ($b_j : 1 \rightarrow 0 \rightarrow 1 \rightarrow 0 \ldots$). Given the considerable quantity of calculations to be carried out for a move, we avoid immediately dropping a BS that has just been selected. To do this, a simple frequency-based mechanism is used:

Let *Freq(i)* be the frequency of a move mv(i) (i.e. the number of times the BS b_i is selected in the partial solution), then the number of iterations during which an element b_i should not be reset to 0 is equal to *Freq(i)*. The number is called tabu tenure of the move mv(i).

In order to implement the tabu list, a vector Tabu of |ß| elements is used. As suggested in Glover and Lugana (1997), each element Tabu(i), i.e. the tabu tenure of mv(i) ($1 \leq i \leq |ß|$) records $Freq(i) + t$ where t is the number of iterations when mv(i) is carried out. In this way, it is quite easy to know, at a later iteration t', if a mv(i) is allowed or not: if there exists $j \in J_i = \{i_1 \ldots i_n\}$ such that $Tabu(j) > t'$ then mv(i) is a forbidden move, otherwise, mv(i) is a possible move.

The tabu status of a move mv(i), such that $s' = s + mv(i)$, is canceled if s' has a better quality than s, i.e. $\xi(s') > \xi(s)$. This condition corresponds to a simple, yet important technique called "aspiration criteria."

7.5. *Diversification*

During the normal search process, the tabu algorithm chooses, at each iteration, one best move among all possible moves. This process is stopped and a diversification phase is triggered if no improved configuration is found during a fixed number of iterations. To do this, we re-calculate the weight of each STP in the following way:

If the STP is already covered by a BS, its weight equals 1, otherwise, the weight equals $1 + |ST|$. One then replaces the evaluation function ξ by ξ' (Section 7.2.). The evaluation function is changed in order to focus the search on the uncovered STP. This mechanism allows the search process to escape from a local optimum.

The number of iterations that trigger a diversification is relatively small, because one does not want to carry out too many non-improving moves, which require many calculations. This number is determined automatically using a simple idea. When our algorithm arrived at a local optimum, it selected |s*| BS, |s*| being the number of elements with 1 in s*. We consider that if it carries out, from this point, |s*| moves without improvement then it is necessary to diversify the search.

Let us notice that during diversification, the value of $c_0'(s)$ does not represent the real coverage ensured by the configuration s. The real coverage $c_0(s)$ is kept up to date during the diversification.

7.6. General algorithm

The TS algorithm is composed of two iterative phases: search by exploitation and diversification. The algorithm's skeleton is shown below:

```
                          Tabu search
Begin   s ← (0...0)
        s* ← s
Search:
        i ← 0
        while ( i ≤ |s*|) do
                search for a non tabu move i with the best Δξ
                s ← s + mv(i)
                if ξ(s) > ξ(s*) then
                        s* ← s
                        i ← 0
                else
                        i ← i + 1
Diversification:
        set the current solution to the best one: s ← s*
        set the best local solution d* to s*: d* ← s*
        compute STP weight (§7.6.)
        i ← 0
        while ( i ≤ |s*|) do
                search for a non tabu move i with the best Δξ'
                s ← s + mv(i)
                if ξ'(s) > ξ' (d*) then
                        i ← 0
                else
                        i ← i + 1
                if ξ(s) > ξ (d*) then
                        d* ← s
        if ξ(d*)>ξ(s*)  then
                s* ← d*
                s ← s*
                go to Search
end
```

The tabu algorithm stops when a diversification is not able to improve the solution with which the diversification starts. The algorithm returns the best solution s* found during the search. This TS algorithm requires no parameter to tune. Note that, if we are only interested in satisfying constraints, a stop condition may be added when the value of the ξ component c_0 (i.e. coverage) is equal to $|ST|$.

8. Post optimization

Generally speaking, the post-optimization phase can be used to optimize any objective (the noise level, the total traffic supported ...) or to enhance constraint satisfaction in case of constraint violation. The basic idea of the post-optimization phase is to improve a solution by fine tuning some antenna parameters.

As discussed in this paper, it is very difficult to satisfy the coverage and OCC constraints simultaneously. The proposed approach satisfies the OCC first and tries to satisfy the coverage constraint during tabu optimization. Typically, tabu optimization alone can result in coverage greater than 99%. If a 100% coverage is not reached, we use the following post optimization technique to cover the remaining 1% of STP.

The principle of this post optimization process is simple: if one slightly increases the power of some BS selected in such a solution s*—to almost the feasibility level—we should be able to obtain the total coverage of the STP, without violating the other constraints. For this purpose, we seek the closest BS bmin of the uncovered STP (in terms of signal power):

$$\delta \mathrm{Cd}_{\mathrm{b,p}} = \mathrm{Sq} - \mathrm{Cd}_{\mathrm{b,p}} (\text{p is not covered so } \delta\mathrm{Cd} > 0)$$
$$\mathrm{bmin} = \mathrm{b} \in BS1 \; / \; \delta\mathrm{Cd} \text{ is minimum}$$

If the power (Ps) of bmin verifies the relation, $\mathrm{Ps} + \delta\mathrm{Cd} \leq 55$ dBm, then one can increase the power of this BS and repeat the operation. This simple process allows us to satisfy the coverage constraint in most cases.

Let us notice that for the post optimization phase, one may use other antenna parameters instead of power. Moreover, this kind of tuning may be easily applied to improve an existing network.

We have developed other techniques for improving objectives, such as traffic hold and noise level. For the purpose of simplification, these techniques are not presented here.

9. Experimentation and numerical results

9.1. Data sets

Computational experiments are carried out on two large and realistic data sets corresponding to two different types of networks: an urban network and a highway network. These test sets were generated by the CNET, which is France Telecom's research laboratory, by using a very

Table 6. Characteristics of the 2 data sets.

	Service	Area width	Area length	Mech size	RTP	STP	TTP	Trafic (Erlang)	sites
Unban Network	8 watt outdoor	46,5 km	45,8 km	200 m	56792	17393	6652	2988,20	568
Highway Network	8 watt outdoor	39 km	168,8 km	200 m	164580	29954	4967	3210,94	250

powerful engineering tool called Parcell. Each data set is described by a file containing the coordinates of the candidate sites, a huge propagation loss matrix calculated using a radio propagation model, and other relevant information concerning the working area, antennas, etc. The data sets are quite large, since each one requires more than 200 MB of memory. The main characteristics of these data are given in Table 6.

We notice that the urban network has fewer STP and more candidate sites than the highway network. In addition, the urban network has a more homogeneous distribution of traffic. A priori this first problem would be thus less difficult than the highway network in terms of satisfying the coverage constraints and optimizing the ability to handle traffic. Of course this analysis does not take into account the propagation loss matrix, which is determining for resolving the APP, however, it does give a first classification of these problems.

9.2. *Computational results*

To solve these two problems, we followed the previously presented resolution procedure. First, the pre-processing algorithm is run to obtain the set ß of BS satisfying the OCC constraints. This phase gives us typically 200,000 to 400,000 BS and takes about 4 hours on an ULTRA SPARC 30 with 512 MB of RAM. Then, the tabu optimization algorithm is executed to find feasible solutions satisfying the handover and coverage constraints. This phase is the most time consuming and takes about 24 to 48 hours to carry out 2,000 to 4,000 iterations. Finally, a post optimization algorithm is used to further improve the solution found in the second phase or to enhance the coverage constraint if needed. This last step takes about 10 minutes, and thus is very fast compared with the first two steps.

There are several ways to obtain different solutions for a network. For example, one may run the tabu algorithm several times with the same ß set. One may also use the pre-processing algorithm to produce different ß sets by varying the filtering criteria used. Table 7 shows three feasible, non-dominate solutions for each network, which are obtained with different ß sets.

Table 7 shows the values of the four objectives. Columns 3 to 6 represent the number of omni-directional, large directional, and small directional antennas, and the number of base stations.

Appendixes 1 and 2 offer graphic representations of two solutions in Table 7, with each color representing a cell of a BS. These figures allow us to have a rough idea

Table 7. Feasible solutions for an urban network and a highway network.

	Sites	OMNI	LD	SD	BS	Noise	Traffic hold		Traffic yeild
Urban network	60	0	29	41	70	215135,10	2239,96	75%	62%
	63	0	41	44	85	274504,5	2339,14	78%	86%
	94	0	0	154	154	216460,3	2988,12	100%	78%
Highway network	58	1	35	67	103	331227,7	2251,50	70%	72%
	78	0	28	128	156	385918,8	2889,55	90%	72%
	86	0	39	78	117	278481,4	2797,75	87%	79%

Table 8. Unfeasible configurations for an urban network and a highway network.

	OCC	Sites	OMNI	LD	SD	BS	Noise	Traffic hold		Traffic yeild
Urban network	2	63	0	0	135	135	344874,45	2940,41	98%	73%
	2	89	0	134	0	134	362020,80	2953,00	99%	80%
Highway network	9	79	0	0	118	118	316644,29	3171,19	99%	80%
	10	110	0	164	0	164	358675,00	3210,91	100%	78%

about the topology of the solutions found. For example, we observe that the cells are quite homogeneous, which is considered to be a desirable property of a network. Appendixes 3 and 4 give the two corresponding solutions in detail. From these detailed tables, we observe that the values of the "power" of antennas are rather close to the high value part. The repartitioning of the "tilt" values is almost homogeneous. We also observe that there are very few omnidirectional antennas. This may be explained by the fact that there is neither constraint nor objective on the number of BS, and several directional antenas can ensure the coverage of an omnidirectional antenna, with much better tuning possibilities.

In a similar way, for each network Table 8 presents two unfeasible solutions, where the OCC constraint is relaxed (number of OCC indicated in the second column). The main purpose of these results is to show the flexibility of the proposed approach. By comparing the results of Tables 7 and 8, we observe that the violation of the OCC constraint increases the noise level. This observation constitutes an empirical justification of the importance of the OCC constraint.

9.3. Comments and discussions

Let us now make some comments about these results and the proposed approach. The first comment concerns the feasibility of solutions for these networks. Given the high complexity of the data used and the way the data has been generated, it was unknown whether any feasible solution existed satisfying all the constraints of the model defined in section 3. It should be noted that among the three constraints, the OCC proved to be particularly difficult to manage. Indeed, we failed to satisfy this constraint with penalty-based approaches. The technique presented in section 6 for handling the OCC constraint proved to be much more powerful.

The second comment concerns the diversity of the solutions found. It is well known that for a multi-objective optimization problem, it is important to have a large number of diversifying or different non-dominate solutions. The experimentation shows that the proposed approach can produce many non-dominate solutions, thanks to its different solving phases.

The third comment concerns the quality of the solutions found. This is a difficult issue, because there is no reference available concerning this matter. However, we know that radio engineers even with the help of the above-mentioned engineering tool, Parcell, found no feasible solution. Compared with such solutions, even without taking into account the factor of feasibility, the results produced by the approach presented in this paper are much better in terms of service quality. Indeed, the noise level is much lower than in hand-made solutions.

Finally, we would like to insist upon the flexibility of the proposed approach. The proposed approach can be used naturally in an interactive environment, which is often necessary for network design. In addition, it can be easily adapted to other models of the APP. Indeed, the model used in this work corresponds to a particular scenario; the constraints and objectives may be exchanged in other models. For example, the coverage constraint may be considered instead as an objective to be maximized. Similarly, the OCC constraint can also be bracketed in order to minimize the extra-connected components. It is easy to see that the proposed approach can be applied directly in these situations.

10. Conclusion

The heuristic aproach we propose in this paper constitutes one of the first studies dealing with antenna positioning and optimization of large and real size networks.

The proposed approach is composed of three sequential phases: a pre-processing phase based on a filtering principle, an optimization phase using tabu search, and a post optimization phase based on fine tuning. The pre-processing phase is parameterized allowing us to generate a variety of reduced sets of BS, of interest for devising an ultimate solution. The tabu algorithm is based on a binary representation of the search space, and integrates techniques such as frequency-based tabu list management, and penalty-based diversification. Various techniques are available for post optimization, either to improve the objectives or to enhance constraint satisfaction.

This approach was applied to two large and realistic test data sets, corresponding to an urban network and a highway network. Results obtained on these data sets show that the proposed approach is very promising for antenna positioning and optimization of large networks. This approach proves to be flexible, robust and effective.

This work deals with the construction of networks from the ground up. Another very important and closely related problem concerns the optimization of networks already in place. The simplest from of optimization concerns only fine-tuning of antenna parameters: powers, tilt, and azimuth ... In addition to such tunings, one may also need to add new BS or new sites, deleting existing BS or sites. Each of these operations has a possibly different cost. A model of this evolution version of the APP is proposed in Reininger (1997). Adaptations of the approach presented in this paper to this model have been carried out and evaluated on large data sets. Once again, computational results show the effectiveness of the approach for dealing with this kind of network.

From this study, we may conclude that although the general APP is a highly combinatorial and complex application, the problem can be resolved using a heuristic approach. Consequently, it must surely be possible to integrate such optimization approaches into engineering tools for radio network planning. We expect such industrial tools to be built and used by network operators in the near future.

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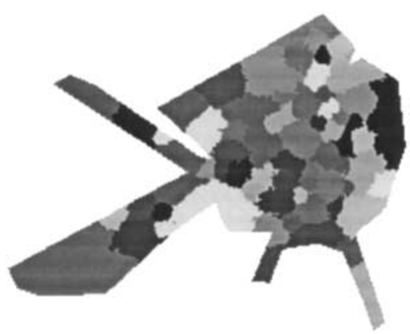
Appendix 1: Urban network cells



Appendix 2: Highway network cells

Appendix 3: TS output for urban network = 70 BS

Site	Antenna	Tilt	Azimut	Ps	Site	Antenna	Tilt	Azimut	Ps	Site	Antenna	Tilt	Azimut	Ps
9	LD	−9	340	28	189	SD	0	30	38	393	SD	−15	60	34
17	LD	−9	150	44	194	SD	−6	60	40	393	SD	−12	280	48
39	LD	−6	130	40	211	SD	−3	80	30	396	LD	−6	200	38
40	SD	−12	80	50	235	LD	−3	340	40	404	LD	−15	170	46
40	SD	0	220	54	253	LD	−9	0	44	406	LD	−15	90	48
56	LD	−9	300	54	266	SD	−9	250	50	408	SD	−15	60	50
59	LD	−9	260	34	277	SD	−6	90	44	413	SD	0	300	36
59	LD	0	40	34	309	LD	−6	140	38	414	SD	−12	60	42
66	SD	−9	0	50	313	LD	−6	290	38	415	SD	−15	60	50
78	SD	0	300	46	315	LD	−9	240	42	426	SD	−6	110	38
78	LD	−12	170	48	316	SD	−9	280	50	426	SD	0	330	38
79	SD	−9	0	54	330	LD	−15	120	44	447	SD	−12	350	46
89	LD	0	180	44	338	SD	−9	20	48	449	SD	−6	230	36
92	SD	−12	300	54	344	SD	0	10	34	464	SD	−12	200	52
100	SD	−6	340	42	356	SD	−15	310	48	473	LD	−15	170	40
100	LD	0	80	36	356	LD	−6	180	36	489	SD	−6	60	46
107	SD	−15	300	42	359	SD	−3	60	30	497	LD	−6	50	38
108	SD	−6	80	36	360	LD	−9	50	32	499	LD	−9	300	46
111	SD	−3	160	46	371	LD	−15	160	52	515	SD	−9	260	50
112	SD	−6	0	42	372	SD	−9	150	54	515	SD	−3	120	48
115	SD	−6	110	32	378	SD	−9	30	42	536	LD	−9	350	30
115	LD	−6	0	44	378	SD	−6	190	40	561	SD	−6	180	38
125	LD	−9	280	46	379	LD	−9	90	46	564	SD	−12	240	54
147	LD	−9	10	34										

Appendix 4: TS output for highway network with 99% coverage (a) and modified BS by post optimization with 100% coverage (b)

Site	Antenna	Tilt	Azimut	Ps	Site	Antenna	Tilt	Azimut	Ps	Site	Antenna	Tilt	Azimut	Ps
9	SD	−12	160	46	74	SD	−6	210	28	144	LD	−9	10	34
9	SD	−6	40	36	74	SD	−3	350	42	144	LD	−3	150	30
13	SD	0	220	30	78	LD	−6	300	28	145	LD	−9	330	40
14	SD	−15	160	36	79	SD	−6	110	54	148	SD	−15	290	42
14	SD	−9	20	44	83	SD	−3	230	42	148	LD	−6	90	42
17	**SD**	**−9**	**190**	**34**	87	LD	−9	320	44	149	SD	−6	60	54
17	SD	−9	0	42	88	SD	−6	290	26	154	LD	−6	260	46
18	LD	0	10	34	89	SD	−3	290	50	156	SD	0	40	46
20	SD	−9	20	30	92	SD	−12	250	54	165	LD	−6	230	34
20	**SD**	**0**	**320**	**26**	93	LD	−9	310	44	172	LD	0	290	42

(*Continued on next page*.)

(*Continued*).

Site	Antenna	Tilt	Azimut	Ps	Site	Antenna	Tilt	Azimut	Ps	Site	Antenna	Tilt	Azimut	Ps
21	SD	−12	40	48	94	SD	−9	50	44	174	SD	−9	260	36
21	LD	−15	350	46	95	SD	−12	90	52	175	SD	−15	290	48
22	SD	−15	120	40	95	LD	−9	300	40	175	SD	0	100	30
22	SD	−12	240	52	99	SD	−9	100	38	176	SD	−15	220	46
24	SD	−3	150	26	99	LD	−9	210	50	178	SD	−9	130	32
30	SD	−9	110	48	103	LD	−9	110	32	179	SD	0	310	54
30	LD	−9	320	40	104	SD	−6	50	54	179	LD	−9	50	50
35	SD	−12	140	55	105	SD	−9	310	52	180	LD	−9	110	34
35	SD	−6	220	50	105	LD	−9	190	44	185	SD	−9	170	42
38	LD	−6	120	52	109	SD	−9	70	28	186	SD	−15	80	34
42	SD	−6	250	54	109	LD	−9	270	34	188	LD	−15	230	48
42	**SD**	**0**	**350**	**42**	110	SD	−9	90	46	193	SD	−6	60	46
42	SD	0	90	44	112	SD	−9	70	36	199	SD	−6	60	42
44	SD	−6	280	48	113	LD	−9	170	28	202	LD	−6	40	30
44	SD	−6	230	46	114	LD	−9	10	26	207	LD	−3	130	36
44	SD	−6	110	44	119	SD	−15	10	40	208	LD	−6	240	44
50	SD	−12	310	44	119	SD	−9	300	36	209	LD	−6	340	36
50	LD	−9	100	44	121	SD	0	210	28	213	SD	−15	220	44
51	SD	−3	30	38	123	SD	−6	250	32	214	SD	−6	270	26
53	SD	−9	300	34	127	LD	−9	170	32	214	SD	−6	100	42
60	LD	−9	220	32	128	SD	−9	200	44	216	SD	−12	290	48
62	SD	−6	290	42	128	SD	−3	330	34	216	LD	−6	100	34
62	SD	−6	50	34	128	LD	−15	40	36	221	SD	−9	250	46
63	SD	−6	70	54	137	SD	−9	0	44	222	LD	−9	90	40
64	SD	−6	300	52	139	SD	−15	140	52	**230**	**LD**	**−9**	**230**	**26**
64	LD	−9	100	32	140	SD	−6	20	40	**231**	**SD**	**−3**	**280**	**34**
70	SD	−9	260	54	**142**	**LD**	**−9**	**80**	**42**	241	SD	−15	120	32
70	SD	−6	140	44	**143**	**LD**	**−15**	**40**	**40**	243	SD	−15	240	42
72	SD	−9	190	46	143	LD	−9	250	30	247	SD	−15	30	40

(a)

Site	Antenna	Tilt	Azimut	Ps
17	SD	−9	190	35
20	SD	0	320	27
42	SD	0	350	52
44	SD	−6	230	55
142	LD	−9	80	43
143	LD	−15	40	44
230	LD	−9	230	28
231	SD	−3	280	36

(b)

Notes

1. The formal definition of the notion of cell is given later in Section 3.1.
2. The choice, done in Reininger (1997) and Reininger and Caminada (1998a), is based on the fact that each point STP has 8 neighboring STP.
3. Notice that non-uniformed costs have no incidence for the heuristic approach presented in this paper.

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